



# Emittance growth and beam lifetime limitations due to beam-beam effects in $e^+e^-$ storage ring colliders

Jie Gao

## ► To cite this version:

Jie Gao. Emittance growth and beam lifetime limitations due to beam-beam effects in  $e^+e^-$  storage ring colliders. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, 2004, 533, pp.270-274. 10.1016/j.nima.2004.06.137 . in2p3-00023724

**HAL Id: in2p3-00023724**

**<https://hal.in2p3.fr/in2p3-00023724>**

Submitted on 7 Feb 2005

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Emittance Growth and Beam Lifetime Limitations due to Beam-Beam Effects in $e^+e^-$ Storage Ring Colliders

J. Gao

**Laboratoire de L'Accélérateur Linéaire**

*IN2P3-CNRS et Université de Paris-Sud*

*B.P. 34, 91898 Orsay cedex, France*

*and*

**Stanford Linear Accelerator Center**

*Stanford University*

*2575 Sand Hill Road, Menlo Park, CA 94039, USA*

## Abstract

In this paper we give analytical expressions for the maximum beam-beam parameter and related beam-beam limited beam lifetime in  $e^+e^-$  storage ring colliders. After analysing the performances of existing or existed machines, we make some discussions on the parameter choice for the Super-B factory design.

## 1 Introduction

For about four decades, beam-beam effect has been a subject of scientific research for its limiting nature on the performance of storage ring colliders, and countless publications have been dedicated to it. As a very comprehensive and classical review on beam-beam effect, readers are directed to ref. [1] for detailed information. In this paper we treat the beam-beam limitations from two directions, firstly, from emittance blow-up point of view (see ref. [2], which is modified in this paper), secondly, from the point of view of beam-beam limited beam lifetime (see ref. [3]), and finally, we combine them to a unified theory. Since there are some modifications to ref. [2], we spend more inks in section 2 to clarify emittance blow-up mechanism, and section 3 is devoted to the unified beam-beam effect theory.

In section 4 experimental results obtained in different machines are compared with analytical ones, and finally, in section 5, parameter choice for Super-B factory has been briefly discussed.

## 2 Beam-beam parameter limit coming from beam emittance blow-up

In  $e^+e^-$  storage ring colliders, due to strong quantum excitation and synchrotron damping effects, the particles are confined inside a bunch. The state of the particles can be regarded as a gas, where the positions of the particles follow statistic laws. When two bunches undergo collision at an interaction point (IP, denoted by “\*”) the particles in each bunch will suffer from additional heating. Taking the vertical plane for example, one has beam-beam induced kicks in  $y$  and  $y' = dy/ds$  expressed as:

$$\delta y = -\frac{\sigma_s}{f_y} y \quad (1)$$

$$\delta y' = -\frac{y}{f_y} y \quad (2)$$

$$\frac{1}{f_y} = \frac{2N_e r_e}{\gamma \sigma_{y,*,+} (\sigma_{x,*,+} + \sigma_{y,*,+})} \quad (3)$$

where  $\sigma_s$  is the bunch length,  $N_e$  is the particle number inside the bunch,  $r_e$  is the electron classical radius,  $\sigma_{x,*,+}$  and  $\sigma_{y,*,+}$  are bunch transverse dimensions just before the two colliding bunches overlapping each other, and  $\sigma_{x,*}$  and  $\sigma_{y,*}$  are defined as the transverse dimensions when the two bunches are fully overlapped at IP. The invariant of vertical betatron motion can be expressed as [4]:

$$a_y^2 = \frac{1}{\beta_y^*} \left( y_*^2 + \left( \beta_{y,*} y_*' - \frac{1}{2} \beta_{y,*}' y_* \right)^2 \right) \quad (4)$$

From eqs. 1 and 2 one finds that

$$\delta a_y^2 = \frac{1}{\beta_{y,*}} \left( \frac{\sigma_s}{f_y} \right)^2 y_*^2 \left( 1 + \left( \frac{\beta_{y,*}}{\sigma_s} \right)^2 \right) \quad (5)$$

where  $y_*$  is the vertical displacement of the test particle with respect to the center of the colliding bunch. Due to the gaseous nature of the particles, one has to take an average of all possible values of  $y_*$  according to its statistical distribution function, and from eq. 5 one obtains:

$$\langle \delta a^2 \rangle = \frac{1}{\beta_{y,*}} \left( \frac{\sigma_s \sigma_{y,*}}{f_y} \right)^2 \left( 1 + \left( \frac{\beta_{y,*}}{\sigma_s} \right)^2 \right) \quad (6)$$

The resultant particles' vertical dimension combining the synchrotron radiation and beam-beam effects can be expressed as follows:

$$\sigma_{y,*}^2 = \frac{1}{4} \tau_y \beta_{y,*} \left( Q_y + \frac{1}{T_0 \beta_{y,*}} \left( \frac{\sigma_s \sigma_{y,*}}{f_y} \right)^2 \left( 1 + \left( \frac{\beta_{y,*}}{\sigma_s} \right)^2 \right) \right) \quad (7)$$

where  $T_0$  is the revolution time,  $\tau_y$  is the radiation damping time, and  $Q_y$  is defined according to ref. [4] as  $\sigma_{y,*}^2 = \frac{1}{4} \tau_y \beta_{y,*} Q_y$  with  $\sigma_{y,*}$  being bunch natural vertical dimension at IP. Solving eq. 7, one finds

$$\sigma_{y,*}^2 = \frac{\sigma_{y,*}^2}{\left( 1 - \frac{\tau_y}{4T_0} \left( \frac{e^2 N_e K_{bb,y}}{E_0} \right)^2 \right)} \quad (8)$$

where  $E_0$  is particles' energy, and

$$K_{bb,y} = \frac{\sigma_s}{2\pi \epsilon_0 \sigma_{y,*} (\sigma_{x,*} + \sigma_{y,*})} \left( 1 + \left( \frac{\beta_{y,*}}{\sigma_s} \right)^2 \right)^{1/2} \quad (9)$$

Since  $\sigma_y(s) = \sqrt{\epsilon_y \beta_y(s)}$ , from eq. 8 one gets:

$$\epsilon_y = \frac{\epsilon_{y,0}}{\left( 1 - \frac{\tau_y}{4T_0} \left( \frac{e^2 N_e K_{bb,y}}{E_0} \right)^2 \right)} \quad (10)$$

where  $\epsilon_{y,0}$  is the natural transverse emittance. For a flat bunch ( $\sigma_{y,*} \ll \sigma_{x,*}$ ), from eq. 10 one knows that

$$\sigma_{x,*} \sigma_{y,*} > \left( \frac{3RN_{IP}(e^2 f N_e \beta_{y,*})^2}{8\pi^2 \epsilon_0 m_0 c^2 \gamma^5} \right)^{1/2} \quad (11)$$

Defining

$$H = \frac{\sigma_{x,*} + \sigma_{y,*}}{\sigma_{x,*}\sigma_{y,*}} \quad (12)$$

where  $H$  is a measure of the plasma pinch effect, assuming that  $H$  can be expressed as follows

$$H = \frac{H_0}{\sqrt{\gamma}} \quad (13)$$

and recalling the beam-beam parameter definition:

$$\xi_y = \frac{N_e r_e \beta_{y,*}}{2\pi\gamma\sigma_{y,*}(\sigma_{x,*} + \sigma_{y,*})} \quad (14)$$

where  $\beta_y^*$  is the beta function value at the interaction point,  $\sigma_x^*$  and  $\sigma_y^*$  are the bunch transverse dimensions after the plasma pinch effect, respectively, and finally, by combining eqs. 11, 13 and 14 one gets in general case

$$\xi_y \leq \xi_{y,max,em,flat} = \frac{H_0}{2\pi F} \sqrt{\frac{T_0}{\tau_y \gamma N_{IP}}} \quad (15)$$

or for isomagnetic case

$$\xi_y \leq \xi_{y,max,em,flat} = \frac{H_0 \gamma}{F} \sqrt{\frac{r_e}{6\pi R N_{IP}}} \quad (16)$$

where  $H_0 \approx 2845$ ,  $R$  is the local dipole bending radius, and  $F$  is expressed as follows

$$F = \frac{\sigma_s}{\sqrt{2}\beta_{y,*}} \left( 1 + \left( \frac{\beta_{y,*}}{\sigma_s} \right)^2 \right)^{1/2} \quad (17)$$

The subscript *em* in eqs. 15 and 16 denotes the emittance blow-up limited beam-beam parameter. When  $\sigma_s = \beta_{y,*}$  one has  $F = 1$ .

### 3 Beam-beam parameter limit coming from beam-beam induced beam lifetime

In ref. [3] we have derived beam-beam effect limited beam lifetimes for a rigid flat beam

$$\tau_{bb,y,flat} = \frac{\tau_y}{2} \left( \frac{3}{\sqrt{2}\pi\xi_y N_{IP}} \right)^{-1} \exp \left( \frac{3}{\sqrt{2}\pi\xi_y N_{IP}} \right) \quad (18)$$

$$\tau_{bb,x,flat} = \frac{\tau_x}{2} \left( \frac{3}{\pi \xi_x N_{IP}} \right)^{-1} \exp \left( \frac{3}{\pi \xi_x N_{IP}} \right) \quad (19)$$

and a rigid round beam

$$\tau_{bb,y,round} = \frac{\tau_y}{2} \left( \frac{4}{\pi \xi_x N_{IP}} \right)^{-1} \exp \left( \frac{4}{\pi \xi_x N_{IP}} \right) \quad (20)$$

From eqs. 18 and 19 one finds that for the same  $\tau_{y,bb,flat}/\tau_y$ ,  $\tau_{x,bb,flat}/\tau_x$ , and  $\tau_{y,bb,round}/\tau_y$ , one has  $\xi_{x,flat} = \sqrt{2}\xi_{y,flat}$ , and  $\xi_{y,round} = \frac{4\sqrt{2}}{3}\xi_{y,flat} = 1.89\xi_{y,flat}$ .

Now taking into account of the emittance blow-up effect due to beam-beam interactions, in a heuristic way, one gets

$$\tau_{bb,y,flat} = \frac{\tau_y}{2} \left( \frac{3\xi_{y,max,em,flat}}{\sqrt{2}\pi\xi_{y,max,0}\xi_y N_{IP}} \right)^{-1} \exp \left( \frac{3\xi_{y,max,em,flat}}{\sqrt{2}\pi\xi_{y,max,0}\xi_y N_{IP}} \right) \quad (21)$$

and

$$\tau_{bb,y,round} = \frac{\tau_y}{2} \left( \frac{3\xi_{y,max,em,round}}{\sqrt{2}\pi\xi_{y,max,0}\xi_y N_{IP}} \right)^{-1} \exp \left( \frac{3\xi_{y,max,em,round}}{\sqrt{2}\pi\xi_{y,max,0}\xi_y N_{IP}} \right) \quad (22)$$

with

$$\xi_{y,max,em,round} = 1.89\xi_{y,max,em,flat} \quad (23)$$

where  $\xi_{y,max,0}$  is rigid beam case limiting value. Taking  $\xi_{y,max,0} = 0.0447$  means that we quantify the term "beam-beam limit" for the beam-beam limited beam lifetime being one hour at  $\tau_y = 30$  ms.

## 4 Comparison of some machine performances with respect to theoretical estimations

We start with the machine parameters [5] shown in Table 1 where the beam energy ranges from half GeV (DAFNE) up to almost hundred GeV, LEP-200, among which there are two machines making the collisions with non zero crossing angle, i.e., DAFNE and KEK-B. Using Table 1 and eq. 15 and assuming  $F = 1$ , the theoretical head-on collision beam-beam parameter limits are given in Table 2. The experimentally achieved maximum beam-beam parameters are shown in Table 2 also with or without crossing angle.

Machine	$N_{IP}$	Energy (GeV)	$\gamma$	$\tau_y$ (ms)	$T_0$ ( $\mu$ s)	$\Phi_{Piwin}$
DAFNE	1	0.51	$10^3$	36	0.325	0.22
BEPC	1	1.89	$3.7 \times 10^3$	28	0.8	0
PEP-II(L)	1	3.12	$6.12 \times 10^3$	62	7.33	0
KEKB(L)	1	3.5	$6.86 \times 10^3$	43	10.05	0.69
KEKB(H)	1	8	$1.57 \times 10^4$	46	10.05	0.69
PEP-II(H)	1	8.99	$1.76 \times 10^4$	37	7.33	0
LEP-100	4	45	$8.82 \times 10^4$	38	88.9	0
LEP-200	4	80.5	$1.58 \times 10^5$	5	88.9	0

Table 1: The machine parameters

Machine	$\xi_{y,max,theory}$	$\xi_{y,max,exp}$
DAFNE	0.043	0.02
BEPC	0.04	0.04
PEP-II(L)	0.063	0.06
KEKB(L)	0.084	0.069
KEKB(H)	0.053	0.052
PEP-II(H)	0.048	0.048
LEP-I	0.037	0.033
LEP-II	0.076	0.079

Table 2: The theoretical maximum and experimentally achieved beam-beam parameters

The agreement between the two sets of values is quite well. Two machines, KEK-B factory and DAFNE, which have finite crossing angles, deserve further analyses. From Table 2 one finds that with Piwinski crossing angle  $\Phi = 0.69$  the experimentally achieved KEK-B low energy ring's (positron) maximum vertical beam-beam parameter is 20% lower than that of head-on collision. On the contrary, the maximum achieved vertical beam-beam parameter of high energy ring seems not to have been affected by the large crossing angle. As for DAFNE, according to the theoretical analysis method described in ref. [6], it seems that the experimentally achieved rather low

vertical beam-beam parameter (0.02) should not be due to Piwinski angle of  $\Phi = 0.22$ , but might be due to bunch lengthening effect [7] in addition to nonlinear electron cloud effect [8] (the electron cloud density  $\rho_{ec} \approx 5 \times 10^{12}$ ).

## 5 Some discussions concerning Super-B factory designs

Super-B factory, or so-called next generation B factory, should have a luminosity larger than  $10^{36}/\text{cm}^2/\text{s}$  [9][10], which could be expressed as follows [9]:

$$L_{max} = 2.17 \times 10^{34} (1 + r) \xi_{y,max} \frac{E_0(\text{GeV}) N_b I_b(\text{A})}{\beta_{y,*}(\text{cm})} [\text{cm}^{-2} \text{s}^{-1}] \quad (24)$$

where  $r = \sigma_{y,*}/\sigma_{x,*}$ ,  $N_b$  is the number of bunches inside a beam, and  $I_b$  is the average current of a bunch. Since  $\xi_y$  depends  $I_b$ , beam transverse dimensions at IP, and  $\beta_{y,*}$ , the critical thing in pushing  $\xi_y$  to its maximum value  $\xi_{y,max}$  expressed in eq. 15 for flat beam and eq. 23 for round beam is to choose carefully  $I_{b,max}$  at which  $\xi_{y,max}$  is reached that the bunch length  $\sigma_s(I_{b,max})$  should be almost same as  $\beta_{y,*}$ , and the rest thing is to push  $N_b$  to realize the required luminosity. In the following, based on the beam-beam effect theory developed above, we will discuss the initial parameters for a  $10^{36}$  B-Factory proposed by Seeman in ref. [9]. Given  $E_0 = 3.1$  GeV,  $I_b N_b = 19.2$  A,  $N_b = 3492$ ,  $\beta_{y,*} = 0.32$  cm,  $\sigma_{s,0} = 3.5$  mm,  $\xi_y = 0.14$ ,  $r = 1$ , and  $\tau_y = 63$  ms, from eqs. 15, 22, 23 (where  $\xi_{y,max,em,flat} = 0.06$  is used), and 24, one finds  $L_{max} = 1.13 \times 10^{36}$  and the beam-beam limited beam lifetime  $\tau_{bb,y,round} = 9$  minutes, which agree quite well with Seeman's estimation [9]. As stated above, one of the critical points is that the bunch length after lengthening effect should be of almost the same value of the vertical beta function at IP. Since the new bunch current and the zero current bunch length shown above have values of a factor of 3.7 higher and factor of 3 lower compared with those of the actual PEP-II LER corresponding values ( $N_b I_b = 1.55$  A,  $N_b = 1034$ , and  $\sigma_{s,0} = 1$  cm) [5], the real bunch length after bunch lengthening could reach  $\sigma_{s,I_b} = 11$  mm (assuming that the vacuum chamber keeps the same as that of actual PEP-II LER [7]), which is more than thrice of  $\beta_{y,*}$  value. The problem of the bunch lengthening, therefore, should be considered in choosing the parameters right from the beginning as a critical issue.



## 6 Conclusions

In this paper we have presented analytical expressions for the maximum beam-beam parameters and the corresponding beam-beam limited beam lifetimes for flat and round colliding beam cases. By applying these analytical formula to the  $10^{36}$  B-Factory parameters given in ref. [9], one finds a similar beam-beam effect limited beam lifetime, however, it is found also that if the same vacuum chamber as PEP-II LER is to be used, the real bunch length with the bunch lengthening effect is thrice larger than the vertical beta function at IP. Therefore, bunch lengthening for Super-B Factory is a critical issue.

## 7 Acknowledgements

The author thanks J. Seeman and G. Wormser for inviting him to work on PEP-II at SLAC, and he thanks also the constant supports from B. D’Almagne, director of LAL, and T. Garvey.

## References

- [1] J.T. Seeman, "Observations of the beam-beam interaction", Lecture Notes in Physics, Vol. 247 (Proceedings of joint US-CERN school on particle accelerators, Sardinia, 1985), p. 121.
- [2] J. Gao, "Analytical expression for the maximum beam-beam tune shift in electron storage rings", **Nucl. Instr. and Methods**, **A413** (1998), p. 431.
- [3] J. Gao, "Analytical estimation of the beam-beam interaction limited dynamic apertures and lifetimes in  $e^+e^-$  circular colliders", **Nucl. Instr. and Methods**, **A463** (2001), p. 50.
- [4] M. Sands, "The physics of electron storage rings, an introduction", SLAC-121.
- [5] Beam Dynamics Newsletter, No. 31, August 2003, edited by Y. Funakoshi, Appendix B (and some other sources).

- [6] J. Gao, "Analytical estimation of the effects of crossing angle on the luminosity of an  $e^+e^-$  circular collider", **Nucl. Instr. and Methods**, **A481** (2001), p. 756.
- [7] J. Gao, "On the single bunch longitudinal collective effects in electron storage rings", **Nucl. Instr. and Methods**, **A491** (2002), p. 1.
- [8] J. Gao, "Positron beam lifetime limited by the combined beam-beam and electron-cloud effects in  $e^+e^-$  storage ring colliders", LAL-SERA-2003-93 (2003).
- [9] J.T. Seeman, "Higher luminosity B-factories", SLAC-PUB-9431, August 2002.
- [10] J.T. Seeman, "Higher luminosity B-factories", Talk given at 30th Advanced ICFA Beam Dynamics Workshop on High Luminosity  $e^+e^-$  Collisions, Oct. 13-16, 2003, Stanford, California, USA.